

Estimate for the $X(3872) \rightarrow \gamma J/\psi$ decay width

Yubing Dong^{1,2,3}, Amand Faessler¹, Thomas Gutsche¹, Valery E. Lyubovitskij^{1*}

¹ *Institut für Theoretische Physik, Universität Tübingen,
Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

² *Institute of High Energy Physics,
Beijing 100049, P. R. China*

³ *Theoretical Physics Center for Science Facilities (TPCSF),
CAS, Beijing 100049, P. R. China*

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The $X(3872)$ resonance is considered as a hadronic molecule, a loosely-bound state of charmed D^0 and D^{*0} mesons, since its mass is very close to the $D^{*0}\bar{D}^0$ threshold. Assuming structure and quantum numbers of $X(3872)$ as $(D^0\bar{D}^{*0} - D^{*0}\bar{D}^0)/\sqrt{2}$ and $J^{PC} = 1^{++}$, we calculate the $X(3872) \rightarrow \gamma J/\psi$ decay width using a phenomenological Lagrangian approach. We also estimate the contribution of an additional $c\bar{c}$ component in the $X(3872)$ to this decay width, which is shown to be suppressed relative to the one of the molecular configuration.

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* On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia

I. INTRODUCTION

During the last years several new meson resonances, whose properties cannot be simply explained and understood in conventional quark models, have been observed in different experiments. The $X(3872)$ is one of such new charmonium states with mass $m_X = 3871.4 \pm 0.6$ MeV and a narrow width of $\Gamma_X < 2.3$ MeV [1]. The first measurement of $X(3872)$ was carried out by the Belle Collaboration 2003 [2] in B -meson decay $B^\pm \rightarrow K^\pm X \rightarrow K^\pm J/\psi \pi^+ \pi^-$. Later the existence of the $X(3872)$ was confirmed in the experiments of the CDF II [3], D0 [4], and BABAR [5] Collaborations. So far, several decay modes of the $X(3872)$ into $\pi^+ \pi^- J/\psi$, $\pi^+ \pi^- \pi^0 J/\psi$, $D^0 \bar{D}^0 \pi^0$ and $\gamma J/\psi$ have been identified [1], which give some constraints on the quantum numbers of this state. In particular, the decay mode $X(3872) \rightarrow \gamma J/\psi$ implies the positive charge parity $C = +$ of this resonance. The three-body decays $X(3872) \rightarrow \pi^+ \pi^- J/\psi$ and $X(3872) \rightarrow D^0 \bar{D}^0 \pi$ together constrain (or almost fix) the spin-parity quantum numbers of X as $J^{PC} = 1^{++}$.

Several structure interpretations for the $X(3872)$ have been proposed in the literature (for a status report see e.g. Refs. [6, 7, 8]): quarkonium ($c\bar{c}$) [9, 10, 11], tetraquark (“diquark–antidiquark” [12]–[19] and “meson–meson” [19]–[23] configurations), hadronic molecule [24]–[44], quarkonium–molecule mixtures [29, 45], $c\bar{c}g$ hybrids (gluonic hadrons) [46], quarkonium–glueball mixtures [47] or even as a dynamical “cusp” related to the near $D^0 \bar{D}^{*0}$ threshold [48]. As was already stressed before in the context of molecular approaches [24]–[44] the $X(3872)$ can be identified with a weakly-bound hadronic molecule whose constituents are D and D^* mesons. The reason for this natural interpretation is that m_X is very close to the $D^0 \bar{D}^{*0}$ threshold and hence is in analogy to the deuteron — a weakly-bound state of proton and neutron. Note, that the idea to treat the charmonium states as hadronic molecules traces back to Refs. [24, 25]. Originally it was proposed that the state $X(3872)$ is a superposition of $D^0 \bar{D}^{*0}$ and $\bar{D}^0 D^{*0}$ pairs. Later (see e.g. discussion in Refs. [32, 34, 35]) also other structures, such as a charmonium state or even other meson pair configurations, were discussed in addition to the $D^0 \bar{D}^{*0}$ + charge conjugate (c.c.) component. Note, the possibility that the $X(3872)$ is a virtual state is not excluded (see e.g. discussion in Ref. [44, 49]). In Ref. [41] (see also [40, 42, 50]) it was correctly argued that the positive charge parity of the $X(3872)$ corresponds to the following wave function: $|X(3872)\rangle = \frac{1}{\sqrt{2}}(|D^0 \bar{D}^{*0}\rangle - |D^{*0} \bar{D}^0\rangle)$. The possibility of two nearly degenerated $X(3872)$ states with positive and negative charge parity has been discussed in Refs. [18, 40].

This paper focuses on the radiative decay $X(3872) \rightarrow \gamma J/\psi$ using a phenomenological Lagrangian approach based on the molecular $(D^0 \bar{D}^{*0} - D^{*0} \bar{D}^0)/\sqrt{2}$ structure of the $X(3872)$. The first observation of the $X(3872) \rightarrow \gamma J/\psi$ decay mode has been reported by the Belle Collaboration [51]. In particular, the Belle Collaboration indicated the product of branching fractions

$$\text{Br}(B \rightarrow XK) \cdot \text{Br}(X \rightarrow \gamma J/\psi) = (1.8 \pm 0.6 \pm 0.1) \times 10^{-6}, \quad (1)$$

and the branching ratio

$$\frac{\Gamma(X \rightarrow \gamma J/\psi)}{\Gamma(X \rightarrow \pi^+ \pi^- J/\psi)} = 0.14 \pm 0.05. \quad (2)$$

Later on, the decay mode $X \rightarrow \gamma J/\psi$ was confirmed by the BABAR Collaboration [52]. Their result for the product of branching fractions was:

$$\text{Br}(B^+ \rightarrow XK^+) \cdot \text{Br}(X \rightarrow \gamma J/\psi) = (3.3 \pm 1.0 \pm 0.3) \times 10^{-6}. \quad (3)$$

A theoretical analysis of the $X(3872) \rightarrow \gamma J/\psi$ decay has been performed in Refs. [9, 30, 33, 35]. In particular, in Ref. [9] the radiative decays of the $X(3872)$ have been considered in detail in the framework of a possible $1D$ and $2P$ charmonium interpretation. It was found, that the results are very sensitive to the model details and to the quantum numbers of the $X(3872)$. For the assignment $J^{PC} = 0^{++}, 1^{++}$ and 2^{++} the following results for $\Gamma(X \rightarrow \gamma J/\psi)$ have been obtained: 1.5 eV, 11 keV and 37.2 keV, respectively. In Ref. [33] different radiative decays of the $X(3872)$ have been studied using a potential model, where both the charmonium and the molecular interpretation of the $X(3872)$ were considered. In the case of the charmonium picture the conclusion of Ref. [9] related to the strong model dependence of the results was confirmed; the use of different potentials and approximations leads to a significant variation of the $X \rightarrow \gamma J/\psi$ decay rate. Accepting the 1^{++} quantum numbers of the $X(3872)$ and using a potential with Coulomb, linear and smeared hyperfine terms the results for $\Gamma(X \rightarrow \gamma J/\psi)$ were given as 139 keV (without the zero recoil and dipole approximations) and 71 keV (using the same set of approximations as in Ref. [9]). In the case of the molecular interpretation two mechanisms, vector meson dominance (VMD) (in the $\rho J/\psi$ and $\omega J/\psi$ components) and light quark annihilation mechanism (in the neutral and charged $D\bar{D}^*$ components), have been analyzed. Here the $X \rightarrow \gamma J/\psi$ rate is dominated by the VMD mechanism and the prediction for the rate $\Gamma(X \rightarrow \gamma J/\psi) = 8$ keV is smaller than in the charmonium picture, but by coincidence similar to the result of [9]. Therefore, one of the conclusions of Ref. [33] was that a more precise measurement of the $X \rightarrow \gamma J/\psi$ decay properties will shed light

on the internal structure of the $X(3872)$. In Ref. [30] it was argued that the radiative decay $X(3872) \rightarrow \gamma J/\psi$ is dominated by the $D^0 \bar{D}^{*0} / \bar{D}^0 D^{*0}$ components of the $X(3872)$ wave function, when the S -wave $D^0 \bar{D}^{*0}$ scattering length is very large. In Ref. [35] the branching ratio $\text{Br}(X \rightarrow \gamma J/\psi)$ has been related to those for $X \rightarrow \pi^+ \pi^- J/\psi$ and $X \rightarrow \pi^+ \pi^- \pi^0 J/\psi$ using VMD. It was concluded that the prediction for $\text{Br}(X \rightarrow \gamma J/\psi)$ is compatible with the Belle data [51] if the relative phase between the coupling constants of X to $J/\psi \omega$ and $J/\psi \rho$ pairs is small.

In Refs. [53] we developed the formalism for the study of recently observed exotic meson states (like $D_{s0}^*(2317)$ and $D_{s1}(2460)$) as hadronic molecules. In this paper we extend our formalism to the decay $X \rightarrow \gamma J/\psi$ assuming that the X is the S -wave, positive charge parity ($D^0 \bar{D}^{*0} - D^{*0} \bar{D}^0$)/ $\sqrt{2}$ molecule. As for the case of the D_{s0}^* and D_{s1} states, a composite (molecular) structure of the $X(3872)$ meson is defined by the compositeness condition $Z = 0$ [54, 55, 56] (see also Refs. [53]). This condition implies that the renormalization constant of the hadron wave function is set equal to zero or that the hadron exists as a bound state of its constituents. The compositeness condition was originally applied to the study of the deuteron as a bound state of proton and neutron [54]. Then it was extensively used in low-energy hadron phenomenology as the master equation for the treatment of mesons and baryons as bound states of light and heavy constituent quarks (see e.g. Refs. [55, 56]). By constructing a phenomenological Lagrangian including X , J/ψ , D^0 and D^{*0} mesonic degrees of freedom and photons we calculate one-loop meson diagrams describing the radiative $X \rightarrow \gamma J/\psi$ decay. Note, that recently the similar $\gamma J/\psi$ decay mode of the $X(3700)$, which is supposed to be a $D\bar{D}$ bound state, has been considered in [57] using the chiral unitary approach (with coupled-channel dynamics).

In the present manuscript we proceed as follows. First, in Section II we discuss the basic notions of our approach. We discuss the effective mesonic Lagrangian for the treatment of the $X(3872)$ meson as a $D^0 \bar{D}^{*0} - D^{*0} \bar{D}^0$ bound state. In addition, we include the possibility of a $c\bar{c}$ admixture in the $X(3872)$. In Section III we consider the matrix elements (Feynman diagrams) describing the radiative $\gamma J/\psi$ decay of a mixed $X(3872)$ configuration, including the molecular and quarkonia components. We discuss our numerical results and perform a comparison with other theoretical approaches. We show that the contribution of a possible quarkonium component is suppressed relative to the molecular one. Finally, in Section IV we present a short summary of our results.

II. APPROACH

A. Molecular structure of the $X(3872)$ meson

In this section we discuss the formalism for the study of the $X(3872)$ meson interpreted as a hadronic molecule. We consider the $X(3872)$ as a S -wave molecular state with positive charge parity given by the superposition of $D^0 \bar{D}^{*0}$ and $\bar{D}^0 D^{*0}$ pairs as:

$$|X(3872)\rangle = \frac{1}{\sqrt{2}}(|D^0 \bar{D}^{*0}\rangle - |D^{*0} \bar{D}^0\rangle). \quad (4)$$

We adopt the convention that the spin and parity quantum numbers of the $X(3872)$ are $J^{PC} = 1^{++}$, while its mass we write in the form

$$m_X = m_{D^0} + m_{D^{*0}} - \epsilon, \quad (5)$$

where $m_{D^0} = 1864.85$ MeV and $m_{D^{*0}} = 2006.7$ MeV are the D^0 and D^{*0} meson masses, respectively; $\epsilon > 0$ represents the binding energy. Our framework is based on an effective interaction Lagrangian describing the couplings of the $X(3872)$ meson to its constituents:

$$\mathcal{L}_X^M(x) = i \frac{g_X}{\sqrt{2}} X^\mu(x) \int dy \Phi_M(y^2) \left(D^0(x + w_{D^*D} y) \bar{D}_\mu^{*0}(x - w_{DD^*} y) - \bar{D}^0(x + w_{D^*D} y) D_\mu^{*0}(x - w_{DD^*} y) \right), \quad (6)$$

where the correlation function Φ_M characterizes the finite size of the $X(3872)$ meson as a $(D^0 \bar{D}^{*0} - D^{*0} \bar{D}^0)/\sqrt{2}$ bound state. The index M attached to the Lagrangian and the correlation function refers to the “molecular” configuration. In the nonlocal Lagrangian we use the relative Jacobi coordinate y and the center-of-mass (CM) coordinate x . In Eq. (6) we introduce the kinematical parameters $w_{ij} = m_i/(m_i + m_j)$. A basic requirement for the choice of an explicit form of the correlation function is that its Fourier transform vanishes sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We adopt the Gaussian form, $\tilde{\Phi}_M(p_E^2/\Lambda_M^2) \doteq \exp(-p_E^2/\Lambda_M^2)$, for the Fourier transform of the vertex function, where p_E is the Euclidean Jacobi momentum. Here, Λ_M is a size parameter, which characterizes the distribution of the DD^* constituents inside the molecule.

The coupling constant g_X is determined by the compositeness condition [54, 55, 56] (for an application to $D_{s0}^*(2317)$ and $D_{s1}(2460)$ meson properties see Ref. [53].) It implies that the renormalization constant of the hadron wave function

is set equal to zero:

$$Z_X = 1 - (\Sigma_X^M(m_X^2))' = 0. \quad (7)$$

Here, $(\Sigma_X^M(m_X^2))' = g_X^2 (\Pi_X^M(m_X^2))'$ is the derivative of the transverse part of the mass operator $\Sigma_S^{\mu\nu}$, conventionally split into the transverse Σ_X and longitudinal Σ_X^L parts as:

$$\Sigma_X^{M,\mu\nu}(p) = g_\perp^{\mu\nu} \Sigma_X^M(p^2) + \frac{p^\mu p^\nu}{p^2} \Sigma_X^{M,L}(p^2), \quad (8)$$

where $g_\perp^{\mu\nu} = g^{\mu\nu} - p^\mu p^\nu / p^2$ and $g_\perp^{\mu\nu} p_\mu = 0$. The mass operator of the $X(3872)$ is described by the diagram of Fig.1(a).

To clarify the physical meaning of the compositeness condition, be reminded that the renormalization constant $Z_X^{1/2}$ can also be interpreted as the matrix element between the physical and the corresponding bare state. For the case $Z_X = 0$ it follows that the physical state does not contain the bare one and hence it is exclusively described as a bound state of its constituents. As a result of the interaction of the X meson with its constituents, the X meson is dressed, i.e. its mass and its wave function have to be renormalized.

Following Eq. (7) the coupling constant g_X can be expressed in the form:

$$\frac{1}{g_X^2} = \frac{1}{(4\pi\Lambda_M)^2} \int_0^1 dx \int_0^\infty \frac{d\alpha \alpha P(\alpha, x)}{(1+\alpha)^3} \left[\frac{1}{2\mu_{D^*}^2(1+\alpha)} - \frac{d}{dz} \right] \tilde{\Phi}_X^2(z), \quad (9)$$

where

$$P(\alpha, x) = \alpha^2 x(1-x) + w_{D^*D}^2 \alpha x + w_{DD^*}^2 \alpha(1-x), \quad z = \mu_{D^*}^2 \alpha x + \mu_D^2 \alpha(1-x) - \frac{P(\alpha, x)}{1+\alpha} \mu_X^2, \quad \mu_i = \frac{m_i}{\Lambda_M}. \quad (10)$$

Above expressions are valid for any functional form of the correlation function $\tilde{\Phi}_M(z)$.

B. $X(3872)$ meson as mixture of molecule and charmonium components

Following the suggestion (see e.g. discussion in Refs. [32, 34, 35]) that the $X(3872)$ could be a mixture of molecular and other components – charmonium or even other mesonic pairs, we include the $c\bar{c}$ charmonium component in the ansatz for the $X(3872)$ structure. Then Eq. (4) is extended as

$$|X(3872)\rangle = \frac{\alpha}{\sqrt{2}} (|D^0 \bar{D}^{*0}\rangle - |D^{*0} \bar{D}^0\rangle) + \beta |c\bar{c}\rangle, \quad (11)$$

where the mixing coefficients α and β are kept as free parameters. Later on we also present the result for the radiative decay width of the $X(3872)$ in terms of these free parameters. The Lagrangian describing the couplings of the $X(3872)$ to its molecular and charmonium components is written in extension of (6) as:

$$\begin{aligned} \mathcal{L}_X(x) \equiv \mathcal{L}_X^{M+c\bar{c}}(x) = g_X X^\mu(x) & \left(\frac{i\alpha}{\sqrt{2}} \int dy \Phi_M(y^2) \left(D^0(x + w_{D^*D} y) \bar{D}_\mu^{*0}(x - w_{DD^*} y) \right. \right. \\ & \left. \left. - \bar{D}^0(x + w_{DD^*} y) D_\mu^{*0}(x - w_{D^*D} y) \right) + \frac{\beta}{m_c} \int dy \Phi_C(y^2) \bar{c}(x + y/2) \gamma_\mu \gamma_5 c(x - y/2) \right). \end{aligned} \quad (12)$$

Now the index C indicates quantities related to the charmonium configuration. In particular, the correlation function $\Phi_C(y^2)$ characterizes the distribution of charm quarks in the $X(3872)$. We adopt the Gaussian form for $\Phi_C(y^2)$ function with $\tilde{\Phi}_C(p_E^2/\Lambda_C^2) \doteq \exp(-p_E^2/\Lambda_C^2)$, where Λ_C is a free parameter. For dimensional reasons we divide the charmonium component by the constituent quark mass m_c . We also keep a common coupling constant g_X such that we can consider the direct limit for the pure charmonium case: $\alpha \rightarrow 0$ and $\beta \rightarrow 1$.

Application of the compositeness condition (now including both components – molecular and charmonium) constrains the parameters α and β (or their ratio). Now the compositeness condition reads

$$Z_X = 1 - (\Sigma_X^M(m_X^2))' - (\Sigma_X^C(m_X^2))' = 0, \quad (13)$$

where $(\Sigma_M^{M,C}(m_X^2))'$ are the derivatives of the transverse part of the $X(3872)$ mass operator due to the molecular (Fig.1(a)) and charmonium (Fig.1(b)) component.

C. Effective Lagrangian for the radiative decay $X \rightarrow \gamma J/\psi$

The diagrams contributing to the radiative decay $X \rightarrow \gamma J/\psi$ are shown in Fig.2: the $D^0 D^{*0} - D^{*0}$ meson loop diagram [Fig.2(a)] and the one involving the $D^{*0} D^0 - D^0$ meson loop [Fig.2(b)] originate from the molecular DD^* component, while the quark loop diagram [Fig.2(c)] is related to the contribution of the charmonium component. The corresponding phenomenological Lagrangian formulated in terms of the mesons X , J/ψ (in the Lagrangian we denote it by J_ψ), D^0 , D^{*0} (for simplicity we suppress the charged isopartners), charm quarks and the photon, including free and interaction parts, is written as:

$$\mathcal{L}(x) = \mathcal{L}_{\text{free}}(x) + \mathcal{L}_{\text{int}}(x), \quad (14)$$

where

$$\begin{aligned} \mathcal{L}_{\text{free}}(x) = & \sum_{M=X, J_\psi} \frac{1}{2} M_\mu(x) (g^{\mu\nu} [\square + m_M^2] - \partial^\mu \partial^\nu) M_\nu(x) + \bar{c}(x) (i \not{\partial} - m_c) c(x) - \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \\ & + \bar{D}_\mu^{*0}(x) (g^{\mu\nu} [\square + m_{D^{*0}}^2] - \partial^\mu \partial^\nu) D_\nu^{*0}(x) - \bar{D}^0(x) (\square + m_{D^0}^2) D^0(x), \end{aligned} \quad (15a)$$

$$\mathcal{L}_{\text{int}}(x) = \mathcal{L}_X(x) + \mathcal{L}_{J_\psi}(x) + \mathcal{L}_{J_\psi DD}(x) + \mathcal{L}_{J_\psi D^* D^*}(x) + \mathcal{L}_{D^* D \gamma}(x) + \mathcal{L}_{cc\gamma}(x). \quad (15b)$$

Here, $\mathcal{L}_{D^* D \gamma}$ and $\mathcal{L}_{cc\gamma}(x)$ are the electromagnetic $D^{*0} D^0 \gamma$ and $cc\gamma$ interaction Lagrangians:

$$\mathcal{L}_{D^* D \gamma}(x) = \frac{e}{4} g_{D^{*0} D^0 \gamma} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu}(x) \bar{D}_\alpha^{*0}(x) D^0(x) + \text{H.c.}, \quad (16a)$$

$$\mathcal{L}_{cc\gamma}(x) = \frac{2e}{3} A_\mu(x) \bar{c}(x) \gamma^\mu c(x). \quad (16b)$$

The term $\mathcal{L}_{J_\psi}(x)$ describes the coupling of J/ψ to its constituent charm quarks:

$$\mathcal{L}_{J_\psi}(x) = g_{J_\psi} J_\psi^\mu(x) \bar{c}(x) \gamma_\mu c(x), \quad (17)$$

where g_{J_ψ} is the coupling constant.

$\mathcal{L}_{J_\psi D^0 D^0}$ and $\mathcal{L}_{J_\psi D^{*0} D^{*0}}$ are the respective strong interaction Lagrangians

$$\mathcal{L}_{J_\psi DD}(x) = ig_{J_\psi DD} J_\psi^\mu(x) \left(D^0(x) \partial_\mu \bar{D}^0(x) - \bar{D}^0(x) \partial_\mu D^0(x) \right), \quad (18a)$$

$$\mathcal{L}_{J_\psi D^* D^*}(x) = ig_{J_\psi D^* D^*} \left(J_\psi^{\mu\nu}(x) \bar{D}_\mu^{*0} D_\nu^{*0} + J_\psi^\mu(x) \bar{D}^{*0 \nu} D_{\mu\nu}^{*0} + J_\psi^\nu(x) \bar{D}_{\mu\nu}^{*0} D^{*0 \mu} \right), \quad (18b)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $M_{\mu\nu} = \partial_\mu M_\nu - \partial_\nu M_\mu$ is the stress tensor of the vector mesons with $M = D^{*0}, J_\psi$.

The phenomenological strong Lagrangians (18a) and (18b), describing the couplings of J/ψ to $D(D^*)$ mesons, have been intensively discussed in the context of J/ψ physics, e.g. charmonium absorption by light π and ρ mesons, J/ψ production in DD interactions (see e.g. Refs. [58]-[63]) and, recently, in the analysis of $X(3872)$ decays using a phenomenological meson Lagrangian [50]. Besides a sign difference in the definition of the $g_{J_\psi DD}$ and $g_{J_\psi D^* D^*}$ couplings found in the literature, there is also a difference in the structure of the Lagrangian (18b). Here we follow Ref. [59] what concerns the explicit form of the Lagrangians (18a) and (18b) including the sign convention.

At this level we do not include additional, possible form factors at the meson interaction vertices for reasons of simplicity and to have less number of free parameters. Such form factors would lead to a further reduction of the predicted value for the $X \rightarrow \gamma J/\psi$ decay width. The importance of these form factors was mentioned with respect to different aspects of charm physics, e.g. to obtain a suppression of the J/ψ dissociation cross sections [58]. This implies that our result represents an upper limit for the decay width $\Gamma(X \rightarrow \gamma J/\psi)$.

Values for the coupling constants $g_{J_\psi DD}$ and $g_{J_\psi D^* D^*}$ have been previously deduced using constraints of SU(4) flavor, chiral, heavy quark symmetries and in the VMD model (see e.g. discussion in Refs. [58, 59, 60]). The coupling strengths have also been calculated directly using microscopic approaches like QCD sum rules [61], quark models [62, 63], etc. In the present calculation we will use the world averaged values of couplings $g_{J_\psi DD}$ and $g_{J_\psi D^* D^*}$ of [58, 59, 60, 61, 62, 63]:

$$g_{J_\psi DD} = g_{J_\psi D^* D^*} = 6.5. \quad (19)$$

Next we comment on the coupling constant $g_{D^{*0}D^0\gamma}$, where the value is deduced from the data on strong and radiative decays of D^* mesons. We use the central values for the partial decay width $\Gamma(D^{*+} \rightarrow D^0\pi^+)$ and the D^{*0} branching ratios of:

$$\Gamma(D^{*+} \rightarrow D^0\pi^+) = 65 \text{ keV} , \quad \text{Br}(D^{*0} \rightarrow D^0\pi^0) = 61.9\% , \quad \text{Br}(D^{*0} \rightarrow D^0\gamma) = 38.1\% . \quad (20)$$

The strong decay width $\Gamma(D^{*0} \rightarrow D^0\pi^0)$ is deduced by applying isospin invariance, which relates the $D^{*+}D^0\pi^+$ and $D^{*0}D^0\pi^0$ couplings as

$$\Gamma(D^{*0} \rightarrow D^0\pi^0) = \frac{1}{2} \left(\frac{m_{D^{*+}}}{m_{D^{*0}}} \right)^5 \left(\frac{\lambda(m_{D^{*0}}^2, m_{D^0}^2, m_{\pi^0}^2)}{\lambda(m_{D^{*+}}^2, m_{D^0}^2, m_{\pi^+}^2)} \right)^{3/2} \Gamma(D^{*+} \rightarrow D^0\pi^+) = 42.3 \text{ keV} , \quad (21)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källén function.

Then we have the decay width $\Gamma(D^{*0} \rightarrow D^0\gamma)$ which is expressed through the coupling constant $g_{D^{*0}D^0\gamma}$ as

$$\Gamma(D^{*0} \rightarrow D^0\gamma) = \frac{\alpha}{24} g_{D^{*0}D^0\gamma}^2 m_{D^{*0}}^3 \left(1 - \frac{m_{D^0}^2}{m_{D^{*0}}^2} \right)^3 = 26 \text{ keV} . \quad (22)$$

From Eq. (22) we finally predict

$$g_{D^{*0}D^0\gamma} \simeq 2 \text{ GeV}^{-1} . \quad (23)$$

For the mass m_c of the charm quark we choose the value $m_c = m_X/2$. The coupling $g_{J\psi}$ is related to the coupling $f_{J\psi}$ as

$$g_{J\psi} = \frac{2}{3} \frac{m_{J\psi}}{f_{J\psi}} . \quad (24)$$

The quantity $f_{J\psi}$ is defined by the decay width $J/\Psi \rightarrow \gamma \rightarrow e^+e^-$:

$$\Gamma(J/\Psi \rightarrow e^+e^-) = \frac{16\pi}{27} \frac{\alpha^2}{m_{J\psi}} f_{J\psi}^2 \simeq 5.55 \text{ keV} . \quad (25)$$

Fitting the experimental value with $f_{J\psi} = 416.5 \text{ MeV}$ we obtain $g_{J\psi} \simeq 5$. Finally, in our calculation we have the following free parameters: the size parameter Λ_M in the correlation function $\tilde{\Phi}_M$, describing the distribution of the DD^* constituent in the $X(3872)$, the size parameter Λ_C in the correlation function $\tilde{\Phi}_C$, describing the distribution of the charm quarks in the $X(3872)$ and the ratio $R = \beta/\alpha$ of the mixing parameters involving the molecular and quarkonia components.

III. RADIATIVE DECAY $X(3872) \rightarrow \gamma J/\psi$

A. Matrix element and decay width

The matrix element describing the radiative $X(3872) \rightarrow \gamma J/\psi$ decay is defined in general as follows

$$M(X(p) \rightarrow \gamma(q)J/\psi(p')) = e \varepsilon^{mn\rho\sigma} \epsilon_X^\alpha(p) \epsilon_{J\psi}^\mu(p') \epsilon_\rho^\gamma(q) \frac{q_\sigma}{m_X^2} \left(A g_{\mu n} g_{\alpha m} p q + B g_{\mu n} p_m q_\alpha + C g_{\alpha m} p_n q_\mu \right) , \quad (26)$$

where A , B and C are dimensionless couplings, ϵ_X^α , $\epsilon_{J\psi}^\mu$ and ϵ_ρ^γ are the polarization vectors of $X(3872)$, J/ψ and the photon.

The $X(3872) \rightarrow \gamma J/\psi$ decay width is calculated according to the expression:

$$\Gamma(X(3872) \rightarrow \gamma J/\psi) = \frac{\alpha}{3} \frac{P^{*5}}{m_X^4} \left((A+B)^2 + \frac{m_X^2}{m_{J\psi}^2} (A+C)^2 \right) , \quad (27)$$

where $P^* = (m_X^2 - m_{J\psi}^2)/(2m_X)$ is the three-momentum of the decay products.

B. Numerical result and discussion

First, we discuss our results for the case when the $X(3872)$ is a pure molecular state. We find that the values of g_x are fairly stable with respect to a variation of the scale parameter Λ_M . In particular, when varying Λ_M from 2 to 3 GeV the coupling g_x changes from 7.4 to 7.9 GeV. Values for the decay couplings A^M , B^M and C^M in the same interval of $\Lambda_M = 2 - 3$ GeV are:

$$\begin{aligned} A^M &= 2.34 - 3.77, & B^M &= 1.62 - 1.93, & C^M &= 3.58 - 4.15, & \text{at } \epsilon = 0.7 \text{ MeV}, \\ A^M &= 2.40 - 3.85, & B^M &= 1.65 - 1.96, & C^M &= 3.64 - 4.21, & \text{at } \epsilon = 1 \text{ MeV}, \\ A^M &= 2.49 - 3.97, & B^M &= 1.70 - 1.97, & C^M &= 3.74 - 4.30, & \text{at } \epsilon = 1.5 \text{ MeV}, \end{aligned} \quad (28)$$

for various values of the binding energy ϵ . Here the superscript M refers to the molecular picture. In Table 1, we list our results for the decay width $\Gamma(X(3872) \rightarrow \gamma J/\psi)$ at $\epsilon = 0.7, 1, 1.5$ MeV. The range of values for our results is due to the variation of Λ_M from 2 to 3 GeV. Although the resulting decay width is not very sensitive to a change in the binding energy ϵ , the result depends stronger on the variation of Λ_M . The latter result is consistent with the conclusion of Ref. [33], where the $\Gamma(X(3872) \rightarrow \gamma J/\psi)$ decay width is also very sensitive to details of the wave function or finite-size effects. We obviously need more data to constrain our model parameter Λ_M . We therefore consider the present results as an estimate. For comparison we also present the results of Refs. [9, 33]. As was stressed in [33], in the framework of the charmonium picture there is a strong sensitivity to the model details, e.g. to the choice of binding potential, leading to a variation of the predictions from 11 keV [9] to 139 keV [33]. On the other hand, our result is larger than the prediction $\Gamma(X(3872) \rightarrow \gamma J/\psi) = 8$ keV of the other molecular approach [33]. Therefore, a future precise measurement of $\Gamma(X(3872) \rightarrow \gamma J/\psi)$ will be a crucial check for theoretical approaches.

Next, we consider the admixture of a charmonium component in the $X(3872)$. For the following results we fix the binding energy at $\epsilon = 1$ MeV and use the typical value of $\Lambda_C = 2$ GeV. In this case, the coupling constant g_x is given in terms of the coupling g_x^M , calculated in the “molecular limit”, by

$$g_x = g_x^M \frac{1}{\alpha^2 + 0.3\beta^2}, \quad (29)$$

where $g_x^M = 7.57$ GeV at $\Lambda_M = 2$ GeV and 7.63 GeV at $\Lambda_M = 3$ GeV. The relative contribution of the molecular and charmonium component is not sensitive to a variation of the parameter Λ_M . The limits of a pure molecular or charmonium structure are precise with $\alpha = 1, \beta = 0$ or $\alpha = 0, \beta = 1$.

For the mixed configuration the results for the decay couplings A , B and C can be written in terms of the limiting molecular case (A^M, B^M, C^M) and the ratio $R = \beta/\alpha$:

$$\begin{aligned} A &= \frac{A^M}{\sqrt{1 + 0.3R^2}} (1 + 0.364R), \\ B &= \frac{B^M}{\sqrt{1 + 0.3R^2}} (1 + 0.014R), \\ C &= \frac{B^M}{\sqrt{1 + 0.3R^2}} (1 - 0.020R) \end{aligned} \quad (30)$$

at $\Lambda_M = 2$ GeV and

$$\begin{aligned} A &= \frac{A^M}{\sqrt{1 + 0.3R^2}} (1 + 0.228R), \\ B &= \frac{B^M}{\sqrt{1 + 0.3R^2}} (1 + 0.012R), \\ C &= \frac{B^M}{\sqrt{1 + 0.3R^2}} (1 - 0.018R) \end{aligned} \quad (31)$$

for $\Lambda_M = 3$ GeV.

In the next step we simplify the expression for the $X(3872) \rightarrow \gamma J/\psi$ decay width substituting all known parameters and leaving the dependence on the couplings A , B and C :

$$\Gamma(X(3872) \rightarrow \gamma J/\psi) = 1.77 \text{ keV} ((A + B)^2 + 1.562(A + C)^2). \quad (32)$$

Substituting Eqs. (30) and (31) into the expression (32) we obtain the result for $\Gamma(X(3872) \rightarrow \gamma J/\psi)$ in terms of the width $\Gamma^M(X(3872) \rightarrow \gamma J/\psi)$, calculated in the “molecular limit”, and the ratio R of mixing parameters:

$$\Gamma(X(3872) \rightarrow \gamma J/\psi) = \Gamma^M(X(3872) \rightarrow \gamma J/\psi) (1 + 0.304R + 0.025R^2) \quad (33)$$

at $\Lambda_M = 2$ GeV and

$$\Gamma(X(3872) \rightarrow \gamma J/\psi) = \Gamma^M(X(3872) \rightarrow \gamma J/\psi) (1 + 0.227R + 0.013R^2) \quad (34)$$

at $\Lambda_M = 3$ GeV. Again, $\Gamma^M(X(3872) \rightarrow \gamma J/\psi) = 129.8$ keV at $\Lambda_M = 2$ GeV and 239.1 keV at $\Lambda_M = 3$ GeV (see also Table 1). From the final expression we conclude that the contribution of the charmonium component to the $X(3872) \rightarrow \gamma J/\psi$ decay width is suppressed relative to the one of the molecular component.

IV. SUMMARY

In this paper we have considered the $X(3872)$ resonance with $J^{PC} = 1^{++}$ as a hadronic molecule, a loosely-bound state of charmed D^0 and D^{*0} mesons. We also test the possibility of the admixture of a charmonium component. Using a phenomenological Lagrangian approach we have calculated the radiative $X(3872) \rightarrow \gamma J/\psi$ decay width. We have found that the resulting decay width is not very sensitive to a variation of the binding energy ϵ , while it depends on the variation of Λ_M , related to the size of the hadronic molecule. We give a final prediction for $\Gamma(X(3872) \rightarrow \gamma J/\psi)$ in terms of the ratio $R = \beta/\alpha$, involving the mixing parameters of the charmonium and molecular components. We conclude that the contribution of the molecular component dominates the $X(3872) \rightarrow \gamma J/\psi$ decay width.

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Table 1. Decay width of $X(3872) \rightarrow \gamma J/\psi$ in keV.

Approach	$\Gamma(X(3872) \rightarrow \gamma J/\psi)$
$[c\bar{c}]$, Ref. [9]	11
$[c\bar{c}]$, Ref. [33]	71
$[c\bar{c}]$, Ref. [33]	139
[molecule], Ref. [33]	8
Our results	124.8 - 231.3 ($\epsilon = 0.7$ MeV)
	129.8 - 239.1 ($\epsilon = 1$ MeV)
	138.0 - 251.4 ($\epsilon = 1.5$ MeV)

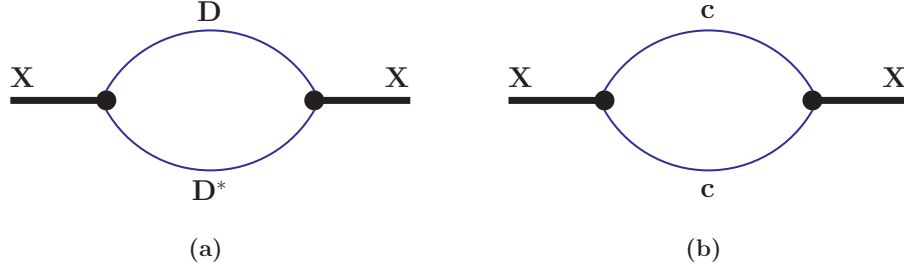


FIG. 1: Diagrams contributing to the mass operator of the $X(3872)$ meson.

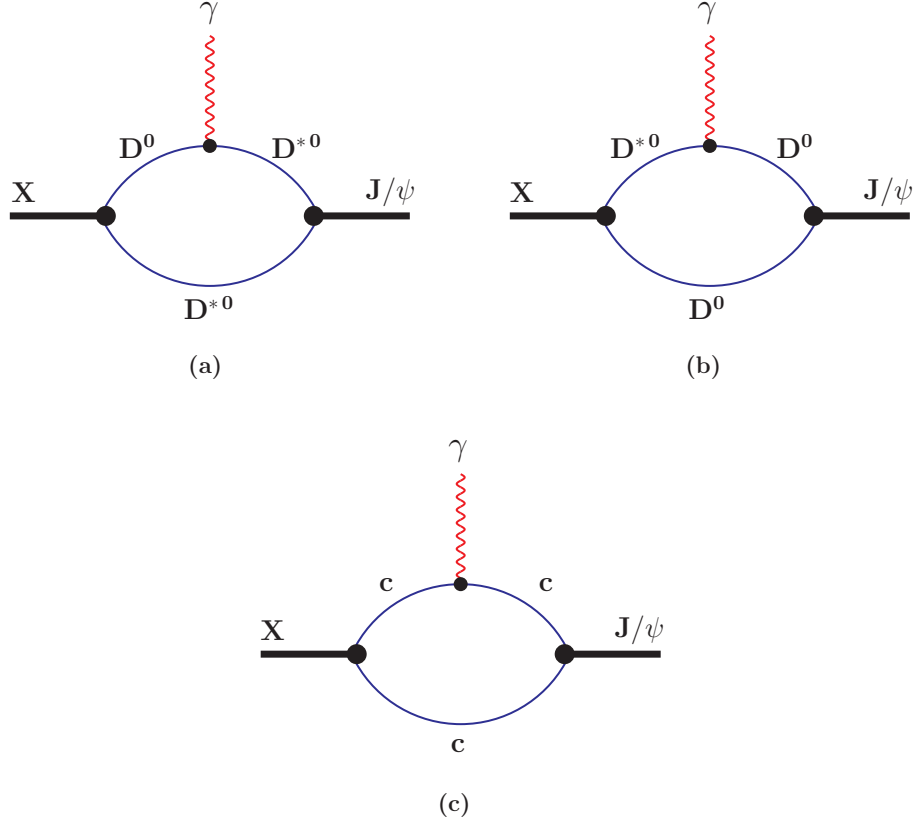


FIG. 2: Diagrams contributing to the radiative transition $X(3872) \rightarrow \gamma J/\psi$.